Asset Bubbles and Credit Constraints

Jianjun Miao[1] Pengfei Wang[2]

[1]Boston University

[2]HKUST

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Our Contribution

- Provide a theory of rational stock price bubbles in firms with endogenous dividends in an infinite horizon model of a production economy.
- A stock price bubble can emerge as long as firms use debt financing subject to sufficiently tight credit constraints endogenously derived from optimal contracts with limited commitment, when other sources of finance are insufficient.
- Critical feature: firm value enters incentive constraints

Key Assumptions

- Firms are heterogeneous in (uninsured) investment opportunities (Kiyotaki and Moore)
- A liquidity mismatch → Capital sales are realized after investment spending.
- Firms face financial frictions:
  - Internal funds are insufficient
  - Debt finance is constrained → based on optimal contracts with limited commitment (Kehoe and Levine, Jermann and Quadrini)
  - Equity finance is limited subject to equity financing costs (BGG, Carlstrom and Fuest, Kiyotaki and Moore..)
  - Other sources are limited: saving, sell capital or other assets
- Why contract frictions do not affect equity?
  - Debt contract → but affect investment, dividends, and equity
  - No contractual frictions between managers and shareholders

Our Story

- Positive feedback loop mechanism: Optimistic beliefs about firm value (bubbles) $\Longrightarrow$ relax credit constraints via incentive constraint $\Longrightarrow$ raise credit limit $\Longrightarrow$ finance more investment $\Longrightarrow$ raise firm value $\Longrightarrow$ justify initial beliefs, support bubbles
  - Different from OLG, Martin and Ventura (2011, 2012), Farhi and Tirole (2012)
  - Inessential for existence of bubble in OLG $\rightarrow$ dynamic efficiency
  - Different from infinite-horizon pure bubble (money), Kiyotaki and Moore, net worth channel $\rightarrow$ cannot generate a stock price bubble
- Stock price bubbles can provide liquidity for investment $\rightarrow$ (money) Kiyotaki and Wright (search), Kiyotaki and Moore (GE)
- Bubbles are self-fulfilling $\rightarrow$ multiple equilibria
- Collapse of stock price bubble $\rightarrow$ stock market crash, recession

Why Important?

- Formalize a positive feedback loop mechanism
  - Understand historical stock market booms and busts, China
  - Transversality conditions, Santos and Woodford (1997)
  - Dynamic efficiency, Abel et al (1989)
  - Rate of return dominance
- Can be integrated into DSGE, quantitative
- Implications for empirical studies
  - Average Q $\neq$ marginal Q
  - TV, Giglio, Maggiori, and Stroebel (2016)
  - Measurement of fundamental value

What is New?

1. The first to model *stock price bubbles* with infinitely lived agents in production economies.
   - Conceptually different from pure bubbles like fiat money
   - Different from commodity money $\rightarrow$ has intrinsic utility value
   - Different mechanism: search, liquidity, net worth
2. The first to present the intuition that the *stock price bubble* pays a dividend (liquidity premium) and hence can grow at less than the discount rate.
   - Different from fiat money (Kiyotaki and Moore, 2005, 08)
3. The first to establish that a *stock price bubble* can relax credit constraints by relaxing incentive constraints, and hence generate value that supports the bubble.
   - Credit constraints are inessential in OLG (Martin and Ventura, Farhi and Tirole)
   - Pyramid schemes

Basic Intuition

- Standard asset pricing under risk neutrality

$$\text{Stock or equity: } V_t = D_t + e^{-r} V_{t+1}$$

- Solution

$$\begin{aligned} V_t &= \underbrace{V_t^*}_{\text{fundamental}} + \underbrace{B_t}_{\text{bubble}} \\ V_t^* &= \sum_{s=0}^{\infty} e^{-rs} D_{t+s} \\ B_t &= e^{-r} B_{t+1} \end{aligned}$$

Issue 1

- If the transversality condition holds

$$\lim_{T\to\infty} e^{-rT} V_{t+T} = 0,$$

then there is no bubble

$$0 = \lim_{T\to\infty} e^{-T} B_{t+T} = B_t.$$

- One solution: finite horizon OLG
- Giglio, Maggiori, and Stroebel (2016) find no evidence

Issue 2

- Bubbly steady state

$$B = e^{-r} B \Longrightarrow r = 0$$

$$V = D + e^{-r} V \Longrightarrow D = 0$$

- A bubbly steady state cannot exist if $D > 0 \rightarrow$ Rate of return dominance!
- A necessary condition $g = 0 \geq r$ (Tirole (1985), Santos and Woodford (1997))

Our Insights

- Asset pricing

$$\text{Stock:} \quad V_t = D_t + e^{-r} V_{t+1}, \quad V_t = Q_t K_t + B_t,$$
$$\text{Bubble:} \quad B_t = e^{-r} B_{t+1} [1 + \underbrace{LIQ_t}_{\text{Liquidity premium}}]$$

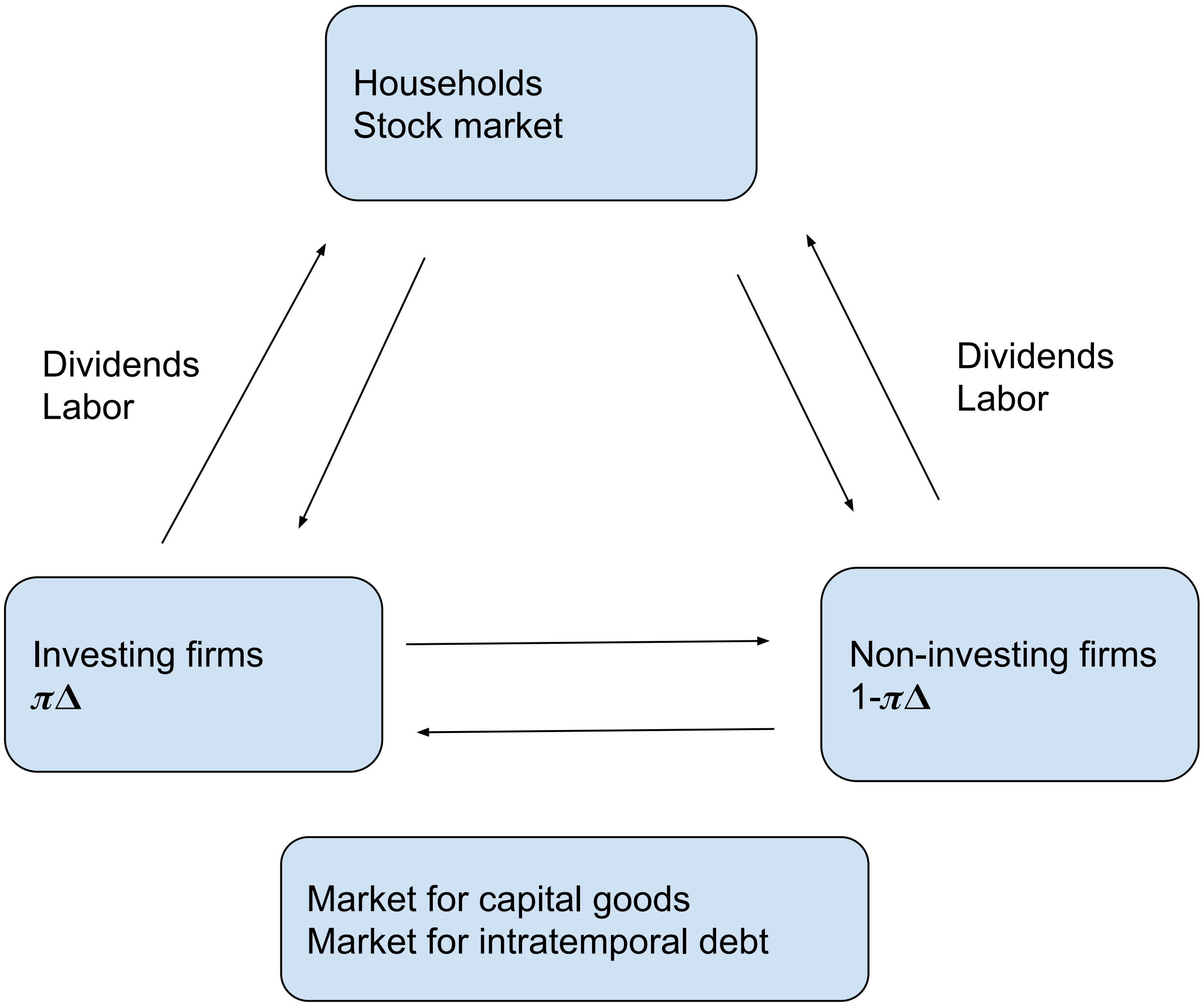
- Firm value enters incentive constraints in optimal contracts
  - Standard borrowing constraints cannot generate a stock price bubble, but can generate a pure bubble
- Stock price bubbles → raise credit limit → raise inv and div
- TV cannot rule out bubbles!
- Bubbly steady state

$$r > 0 = g \Longrightarrow D > 0$$

- Standard asset bubble test rule out OLG bubbles, but not ours!

Baseline Model

- Discrete time $t = 0, \Delta, 2\Delta, \ldots$ Continuous time $\Delta \to 0$
  - Convenient for analysis of local dynamics and stability of steady state
- Identical households and heterogeneous firms $j \in [0, 1]$
- Firms are subject to independent idiosyncratic shocks
- No aggregate uncertainty


Households
Stock market
Dividends
Labor
Dividends
Labor
Investing firms
πΔ
Non-investing firms
1-πΔ
Market for capital goods
Market for intratemporal debt

Asset Markets

- A stock market: only households are shareholders (owners) of all firms
  - Normalize share supply to one
  - No share trading in equilibrium $\longleftarrow$ representative household
  - Firms cannot trade in stock market $\rightarrow$ cross-holdings, direct costs, takeover, fire sale, information
- A market for capital goods: firms buy and sell capital
- An intratemporal debt market: firms lend and borrow among themselves

Household

- Risk neutral (can be relaxed) identical households supply labor inelastically (one unit)
- Trade stocks with unit supply without trading frictions
- Asset pricing for equity

$$V_t^j = D_t^j \Delta + e^{-r\Delta} V_{t+\Delta}^j, \tag{1}$$

- TV

$$\lim_{T \to \infty} e^{-rT} V_{t+T}^j = 0$$

Firms

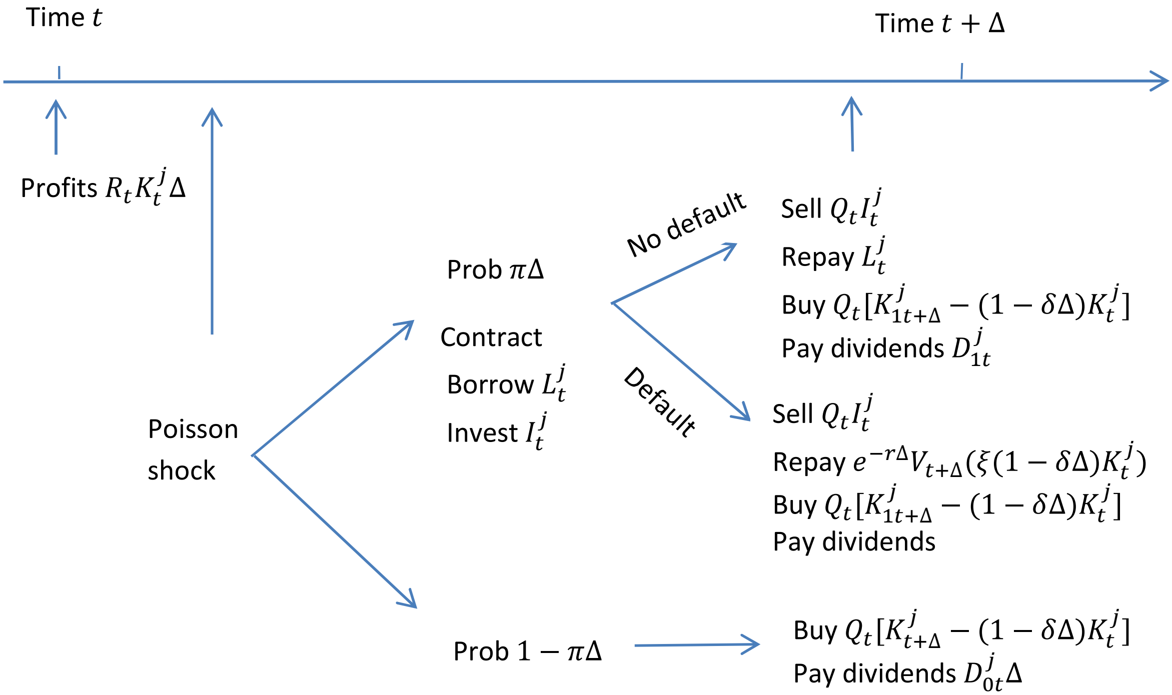
- A continuum of firms indexed by $j \in [0,1]$, with technology

$$Y_t^j = (K_t^j)^\alpha (N_t^j)^{1-\alpha}$$

- Solve static labor choice:

$$\max_{N_t^j} F\left(K_t^j, N_t^j\right) - w_t N_t^j = R_t K_t^j$$

- Investment opportunities arrive independently across firms and over time at Poisson rate $\pi\Delta \longrightarrow$ firm heterogeneity
- I_t^j investment $\longrightarrow I_t^j$ capital goods sold at price Q_t
- Shocks are uninsured (incomplete markets)


Time t
Time $t+\Delta$
Profits $R_t K_t^j \Delta$
Poisson shock
Prob $\pi\Delta$
Contract
Borrow L_t^j
Invest I_t^j
No default
Default
Sell $Q_t I_t^j$
Repay L_t^j
Buy $Q_t[K_{1t+\Delta}^j - (1-\delta\Delta)K_t^j]$
Pay dividends D_{1t}^j
Sell $Q_t I_t^j$
Repay $e^{-r\Delta}V_{t+\Delta}(\xi(1-\delta\Delta)K_t^j)$
Buy $Q_t[K_{1t+\Delta}^j - (1-\delta\Delta)K_t^j]$
Pay dividends
Prob $1-\pi\Delta$
Buy $Q_t[K_{t+\Delta}^j - (1-\delta\Delta)K_t^j]$
Pay dividends $D_{0t}^j\Delta$

Frictions

- Liquidity mismatch: Capital sales $Q_t I_t^j$ are realized after investment spending I_t^j
- Financing options:
  - Internal funds
  - Borrow from other firms through intratemporal debt (limited commitment) $\rightarrow$ can repay after capital sales $Q_t I_t^j > I_t^j$
  - No other sources: saving, sell capital or other financial assets, issue new equity (equity financing costs)

Decision Problem in Discrete Time

$$V_t\left(K_t^j\right) = \max \; (1-\pi\Delta)\left[D_{0t}^j\Delta + e^{-r\Delta}V_{t+\Delta}\left(K_{t+\Delta}^j\right)\right]$$
$$+\pi\Delta\left[D_{1t}^j + e^{-r\Delta}V_{t+\Delta}\left(K_{1t+\Delta}^j\right)\right]$$

subject to

$$D_{0t}^j\Delta + Q_t K_{t+\Delta}^j = R_t K_t^j \Delta + Q_t\left(1-\delta\Delta\right)K_t^j,$$

$$D_{1t}^j + Q_t K_{1t+\Delta}^j + I_t^j + L_t^j = R_t K_t^j \Delta + L_t^j + Q_t\left(1-\delta\Delta\right)K_t^j + Q_t I_t^j,$$

$$I_t^j \le L_t^j + R_t K_t^j \Delta$$

Credit constraints on L_t^j

limit

Decision Problem in Discrete Time

$$V_t\left(K_t^j\right) = \max \ (1-\pi\Delta)\left[D_{0t}^j\Delta + e^{-r\Delta}V_{t+\Delta}\left(K_{t+\Delta}^j\right)\right]$$
$$+\pi\Delta\left[D_{1t}^j + e^{-r\Delta}V_{t+\Delta}\left(K_{1t+\Delta}^j\right)\right]$$

subject to

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$$D_{1t}^j + Q_t K_{1t+\Delta}^j + I_t^j + L_t^j = R_t K_t^j \Delta + L_t^j + Q_t\left(1-\delta\Delta\right)K_t^j + Q_t I_t^j,$$

$$I_t^j \leq L_t^j + R_t K_t^j \Delta$$

Credit constraints on L_t^j

▸ limit

Without Credit Constraints

- Efficient equilibrium
  - $Q_t = 1,\ V_t\left(K_t^j\right) = K_t^j$
  - $R = \alpha(K_E)^{\alpha-1} = r + \delta$
- Limited commitment $\rightarrow$ not feasible

Optimal Contracts with Limited Commitment

- Value of not defaulting $\geq$ Value of defaulting (outside value)
- No default equilibrium!
- Contract I: Borrow against firm value $\rightarrow$ lender can get bubble
- Contract II: Outside value = Diversion value, Gertler, Karadi and Kiyotaki
- Contract III: Outside value = Autarky value, Alvarez and Jermann
- **Essential feature: Firm value is in IC** $\rightarrow$ reputation, speculation...
- Kiyotaki and Moore (1997) or any contract with exogenous credit limit
    - Can generate pure bubbles: Kiyotaki and Moore, Hirano and Yanagawa, Miao and Wang...
    - But cannot generate a stock price bubble!!

Optimal Contract I

- On default, debt is renegotiated: firm has all bargainning power and lender gets the threat value, Jermann and Quadrini
- If firm defaults, the lender threatens to take assets $\xi\left(1-\delta\Delta\right)K_t^j$, reorganize the firm, and obtain going-concern value $e^{-r\Delta}V_{t+\Delta}(\xi\left(1-\delta\Delta\right)K_t^j)$
- Incentive constraint:

$$\underbrace{\ldots - L_t^j}_{\text{Not default}} \geq \underbrace{\ldots - e^{-r\Delta}V_{t+\Delta}(\xi\left(1-\delta\Delta\right)K_t^j)}_{\text{Default}}$$

- $\Longrightarrow$ **Credit constraint**

$$L_t^j \leq e^{-r\Delta}V_{t+\Delta}(\xi\left(1-\delta\Delta\right)K_t^j)$$

- Borrow against firm value
- Different from KM

$$L_t^j \leq \xi Q_t\left(1-\delta\Delta\right)K_t^j$$

Competitive Equilibrium

- Aggregation $K_t = \int_0^1 K_t^j dj$, $I_t = \int_0^1 I_t^j dj$, $N_t = \int_0^1 N_t^j dj$, and $Y_t = \int_0^1 Y_t^j dj$
- Households and firms optimize and markets clear

$$\begin{aligned} N_t &= 1, \psi_t^j = 1, \\ Y_t &= C_t + \pi I_t, \\ \dot{K}_t &= -\delta K_t + \pi I_t \end{aligned}$$

Bubble Solution

- Firm value takes the form:

$$V_t\left(K_t^j\right) = a_t K_t^j + b_t,\ \ V_t(K_t^j) = \underbrace{Q_t K_t^j}_{\text{fundamental}} + \underbrace{B_t}_{\text{bubble}},$$

- Positive feedback loop:

$$\begin{aligned} a_t K_t^j + b_t \quad = \quad & \max_{K_{t+\Delta}^j, K_{1t+\Delta}^j, I_t^j, L_t^j} \quad R_t K_t^j \Delta + Q_t \left(1 - \delta\Delta\right) K_t^j + e^{-r\Delta} b_{t+\Delta} \\ & + \left(1 - \pi\Delta\right)\left[-Q_t K_{t+\Delta}^j + e^{-r\Delta} a_{t+\Delta} K_{t+\Delta}^j\right] \\ & + \pi\Delta\left[\left(Q_t - 1\right) I_t^j - Q_t K_{1t+\Delta}^j + e^{-r\Delta} a_{t+\Delta} K_{1t+\Delta}^j\right] \end{aligned}$$

subject to

$$I_t^j \leq R_t K_t^j \Delta + L_t^j \leq R_t K_t^j \Delta + e^{-r\Delta}\left(a_{t+\Delta}\left(1 - \delta\Delta\right)\xi K_t^j + b_{t+\Delta}\right),$$

- When $Q_t > 1$, the investment and credit constraints bind

Why Stock Price Bubbles?

- Asset pricing

$$b_t = e^{-r\Delta}\left[1 + \pi\Delta\left(Q_t - 1\right)\right] b_{t+\Delta}$$

$$rB_t = \underbrace{\dot{B}_t}_{\text{capital gains}} + \underbrace{\pi(Q_t - 1)B_t}_{\text{liquidity premium}}$$

- TV cannot rule out bubble
- Santos and Woodford condition? $r > 0$, zero economic growth
- Bubble is productive!
- Bubbleless SS is dynamically efficient $\Longrightarrow$ Tirole (1985) condition does not apply

Continuous Time Equilibrium System

Suppose $Q_t > 1$. Then (B_t, Q_t, K_t) satisfy

$$\dot{B}_t = rB_t - B_t\pi(Q_t - 1),$$

$$\dot{Q}_t = (r + \delta)\, Q_t - R_t - \pi\xi Q_t(Q_t - 1),$$

$$\dot{K}_t = -\delta K_t + \pi(\xi Q_t K_t + B_t), \;\; K_0 \text{ given},$$

and the transversality condition:

$$\lim_{T\to\infty} e^{-rT} Q_T K_T = 0, \quad \lim_{T\to\infty} e^{-rT} B_T = 0,$$

- Bubbleless equilibrium: $B_t = 0$
- Bubbly equilibrium: $B_t > 0$

Multiple Equilibria

- Two Steady States
- Both the bubbly steady state (B, Q_b, K_b) and the bubbleless steady state $(0, Q^*, K^*)$ are local saddle points for the nonlinear system for (B_t, Q_t, K_t).
- Bubbleless steady state is indeterminate of degree 1
- Bubbly steady has a unique saddle path
- Stochastic bubbles

Why KM Constraints Fail?

$$
\begin{aligned}
a_t K_t^j + b_t = & \max_{K_{t+\Delta}^j, K_{1t+\Delta}^j, I_t^j, L_t^j} R_t K_t^j \Delta + Q_t (1 - \delta\Delta) K_t^j + e^{-r\Delta} b_{t+\Delta} \\
& + (1 - \pi\Delta) \left[-Q_t K_{t+\Delta}^j + e^{-r\Delta} a_{t+\Delta} K_{t+\Delta}^j \right] \\
& + \pi\Delta \left[(Q_t - 1) I_t^j - Q_t K_{1t+\Delta}^j + e^{-r\Delta} a_{t+\Delta} K_{1t+\Delta}^j \right]
\end{aligned}
$$

subject to

$$
I_t^j \le R_t K_t^j \Delta + L_t^j \le R_t K_t^j \Delta + Q_t (1 - \delta\Delta) \xi K_t^j,
$$

- TV rules out bubbles!

$$
b_t = e^{-r\Delta} b_{t+\Delta}, \quad rB_t = \dot{B}_t
$$

- Need bubble to help finance investment and support value! Martin and Ventura

Difference from Pure Bubbles (Money)

- Money with unit supply, trade at price P_t
- Short sales constraints $M_t^j \geq 0$
- Investing firms sell the bubble asset to non-investing firms
- Raise net worth $\longrightarrow$ bubble can exist, Kiyotaki and Moore (2005, 2008)
- True for any type of borrowing constraints (KM or no borrowing)!

Difference from Pure Bubbles

- Same asset pricing

$$rP_t = \dot{P}_t + \pi \left(Q_t - 1\right) P_t$$

- But stock price bubble is not in a separately traded asset different from stocks

$$V_t \left(K_t^j, M_t^j\right) = Q_t K_t^j + P_t M_t^j.$$

- We have already shown that stock price bubbles cannot exist using KM constraints!
- Given our credit constraints

$$L_t^j \leq V_t \left(\xi K_t^j, 0\right) = \xi Q_t K_t^j + B_t,$$

stock price bubble can coexist with money bubble!

- Different mechanisms

Coexist with Assets with growing rents

- Introduce growth $g \geq 0$
- Asset exogenous rents $X_t = xe^{gt} > 0$
- Asset must be less liquid than stocks

$$M_{1t}^j \geq (1-\zeta)\, M_t^j,$$

- Asset pricing

$$\frac{\dot{P}_t + X_t}{P_t} = r - \pi(Q_t - 1)\zeta > r - \pi(Q_t - 1) = \frac{\dot{B}_t}{B_t}$$

- If $\zeta = 1$, then cannot coexist!

Intertemporal Debt

- Replace intratemporal debt with intertemporal bonds
- Investment is financed by internal funds and debt
- Households cannot borrow or short bonds (cannot issue corporate bonds) and any firm cannot hold or trade other firms' stocks
- Let $L_t^j > (<) 0$ denote debt (saving)
- The lender can seize a fraction ξ of a defaulting firm's existing capital

Intertemporal Debt: Equilibrium

- Firms without investment opportunities save and lend
- Investing firms borrow
- Households do not hold any bonds
- Same equilibrium system
- Bonds and bubbles are perfect substitutes

▸ debt

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debt

Intertemporal Debt: Equilibrium

- Equilibrium interest rate

$$r_{ft} = r - \underbrace{\pi\left(Q_t - 1\right)}_{\text{liquidity premium}} < r$$

$$e^{-r_{ft}\Delta} = e^{-r\Delta}[1 + \pi\Delta(Q_{t+\Delta} - 1)]$$

- Firms are subject to uninsured idiosyncratic investment opportunity shock
- Precautionary saving $\longrightarrow$ low interest rate Aiyagari (1994), Kiyotaki and Moore (2008)
- Households face short-sales constraints on bonds
- Why firms do not trade other firm shares at return $r > r_{ft}$?
    - By assumption, otherwise cross-holdings
    - Not a major financing source in data
    - Costly, fire sale, take over

Liquidity Mismatch

- Suppose firm j can use a fraction of captial sales to finance investment $\lambda Q_t \left(1-\delta\Delta\right) K_t^j$
- Financing constraint

$$I_t^j \leq R_t K_t^j \Delta + L_t^j + Q_t \left[\left(1-\delta\Delta\right) K_t^j - K_{1t+\Delta}^j\right]$$

- Capital

$$K_{1t+\Delta}^j \geq \left(1-\lambda\right)\left(1-\delta\Delta\right) K_t^j$$

- The bubbly steady state exists iff

$$0 < \xi + \lambda < \frac{\delta}{r+\pi}$$

Issue New Equity

- Stock price per share

$$v_t = d_t + e^{-r} v_{t+1}$$

- Define n_t = number of existing shares
  m_t = number of new shares sold at the cum-dividend price v_{t+1}
- Thus, $n_{t+1} = n_t + m_t$
- $V_t = n_t v_t$ total equity value
  $D_t = n_t d_t$ total dividends

Issue New Equity

- We have

$$\begin{aligned} V_t &= D_t + e^{-r} n_t v_{t+1} = D_t + e^{-r} \left[n_{t+1} v_{t+1} - \left(n_{t+1} - n_t \right) v_{t+1} \right] \\ &= D_t - e^{-r} m_t v_{t+1} + e^{-r} n_{t+1} v_{t+1} = \underbrace{\left(D_t - S_t \right)}_{\text{Div}} + e^{-r} V_{t+1} \end{aligned}$$

- $S_t = e^{-r} m_t v_{t+1}$ value of new equity
- Normalize share supply to 1
- Miller and Modigliani (1961 JB)

Issue New Equity

- The external equity financing cost of issuing S_t^j new equity is given by $\varphi(S_t^j)^2/(2K_t^j)$.
- Hennessy and Whited (2007)
- The bubbly steady state exists if and only if

$$0 < \xi(r+\pi) + r/\varphi < \delta.$$

equity

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▸ equity

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▸ equity

Conclusion

- Provide a novel theory of rational bubbles in stocks with endogenous dividends in an infinite horizon model of a production economy.
- Positive feedback loop mechanism via credit channel
- Useful for quantitative study in macro and finance: calibration or estimation (Miao, Wang, and Xu (2015))
- Implications for empirical work
    - Marginal vs average Q
    - Standard bubble test can rule out OLG bubbles, but not ours

Continuous-Time Limit

$$rV_t\left(K_t^j\right) = \max_{\dot{K}_t^j, K_{1t}, I_t^j, L_t^j} D_{0t}^j + \dot{V}_t\left(K_t^j\right) + \pi\left(Q_t - 1\right) I_t^j$$
$$+\pi\left[Q_t K_t^j - Q_t K_{1t}^j + V_t\left(K_{1t}^j\right) - V_t\left(K_t^j\right)\right]$$

subject to

$$D_{0t}^j = R_t K_t^j - Q_t\left(\dot{K}_t^j + \delta K_t^j\right)$$
$$I_t^j \leq L_t^j$$
$$L_t^j \leq V_t(\xi K_t^j).$$

back

Continuous-Time Limit

$$rV_t\left(K_t^j\right) = \max_{\dot{K}_t^j, K_{1t}, I_t^j, L_t^j} D_{0t}^j + \dot{V}_t\left(K_t^j\right) + \pi\left(Q_t - 1\right) I_t^j$$
$$+\pi\left[Q_t K_t^j - Q_t K_{1t}^j + V_t\left(K_{1t}^j\right) - V_t\left(K_t^j\right)\right]$$

subject to

$$D_{0t}^j = R_t K_t^j - Q_t\left(\dot{K}_t^j + \delta K_t^j\right)$$
$$I_t^j \leq L_t^j$$
$$L_t^j \leq V_t(\xi K_t^j).$$

back

Equity Issues

$$V_t\left(K_t^j\right) = \max \quad (1-\pi\Delta)\left[\left(D_{0t}^j - S_{0t}^j\right)\Delta + e^{-r\Delta}V_{t+\Delta}\left(K_{t+\Delta}^j\right)\right]$$
$$+\pi\Delta\left[D_{1t}^j - S_{1t}^j + e^{-r\Delta}V_{t+\Delta}\left(K_{1t+\Delta}^j\right)\right]$$

subject to

$$D_{0t}^j\Delta + Q_t K_{t+\Delta}^j = R_t K_t^j \Delta + Q_t\left(1-\delta\Delta\right)K_t^j + S_{0t}^j\Delta - \frac{\varphi}{2}\frac{(S_{0t}^j)^2}{K_t^j}\Delta,$$

$$D_{1t}^j + Q_t K_{1t+\Delta}^j + L_t^j + I_t^j = R_t K_t^j \Delta + L_t^j + S_{1t}^j - \frac{\varphi}{2}\frac{(S_{1t}^j)^2}{K_t^j}$$
$$+Q_t\left(1-\delta\Delta\right)K_t^j + Q_t I_t^j,$$

$$I_t^j \leq R_t K_t^j \Delta + L_t^j + S_{1t}^j,$$

$$L_t^j \leq e^{-r\Delta}V_{t+\Delta}(\xi\left(1-\delta\Delta\right)K_t^j)$$

back

Equity Issues

$$V_t\left(K_t^j\right) = \max \quad (1-\pi\Delta)\left[\left(D_{0t}^j - S_{0t}^j\right)\Delta + e^{-r\Delta}V_{t+\Delta}\left(K_{t+\Delta}^j\right)\right] \\ +\pi\Delta\left[D_{1t}^j - S_{1t}^j + e^{-r\Delta}V_{t+\Delta}\left(K_{1t+\Delta}^j\right)\right]$$

subject to

$$D_{0t}^j\Delta + Q_t K_{t+\Delta}^j = R_t K_t^j \Delta + Q_t\left(1-\delta\Delta\right)K_t^j + S_{0t}^j\Delta - \frac{\varphi}{2}\frac{\left(S_{0t}^j\right)^2}{K_t^j}\Delta,$$

$$D_{1t}^j + Q_t K_{1t+\Delta}^j + L_t^j + I_t^j = R_t K_t^j \Delta + L_t^j + S_{1t}^j - \frac{\varphi}{2}\frac{\left(S_{1t}^j\right)^2}{K_t^j} \\ +Q_t\left(1-\delta\Delta\right)K_t^j + Q_t I_t^j,$$

$$I_t^j \le R_t K_t^j \Delta + L_t^j + S_{1t}^j,$$

$$L_t^j \le e^{-r\Delta}V_{t+\Delta}(\xi\left(1-\delta\Delta\right)K_t^j)$$

back

Intertemporal Debt

$$V_t\left(K_t^j, L_t^j\right) = \max \quad (1-\pi\Delta)\left[D_{0t}^j\Delta + e^{-r\Delta}V_{t+\Delta}\left(K_{t+\Delta}^j, L_{t+\Delta}^j\right)\right]$$
$$+\pi\Delta\left[D_{1t}^j + e^{-r\Delta}V_{t+\Delta}\left(K_{1t+\Delta}^j, L_{1t+\Delta}^j\right)\right]$$

subject to

$$D_{0t}^j\Delta + Q_t K_{t+\Delta}^j = R_t K_t^j \Delta + e^{-r_{ft}\Delta} L_{t+\Delta}^j - L_t^j + Q_t\left(1-\delta\Delta\right)K_t^j,$$

$$D_{1t}^j + Q_t K_{1t+\Delta}^j + I_t^j = R_t K_t^j \Delta + e^{-r_{ft}\Delta} L_{1t+\Delta}^j - L_t^j + Q_t I_t^j + Q_t\left(1-\delta\Delta\right)K_t^j$$

$$I_t^j \leq R_t K_t^j \Delta + e^{-r_{ft}\Delta} L_{1t+\Delta}^j - L_t^j,$$

$$V_{t+\Delta}(K_{1t+\Delta}^j, L_{1t+\Delta}^j) \geq V_{t+\Delta}\left(K_{1t+\Delta}^j, 0\right) - V_{t+\Delta}(\xi\left(1-\delta\Delta\right)K_t^j, 0),$$

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Intertemporal Debt

$$
\begin{aligned}
V_t\left(K_t^j, L_t^j\right) \quad = \quad & \max \quad (1-\pi\Delta)\left[D_{0t}^j\Delta + e^{-r\Delta}V_{t+\Delta}\left(K_{t+\Delta}^j, L_{t+\Delta}^j\right)\right] \\
& +\pi\Delta\left[D_{1t}^j + e^{-r\Delta}V_{t+\Delta}\left(K_{1t+\Delta}^j, L_{1t+\Delta}^j\right)\right]
\end{aligned}
$$

subject to

$$
D_{0t}^j\Delta + Q_tK_{t+\Delta}^j = R_tK_t^j\Delta + e^{-r_{ft}\Delta}L_{t+\Delta}^j - L_t^j + Q_t\left(1-\delta\Delta\right)K_t^j,
$$

$$
D_{1t}^j + Q_tK_{1t+\Delta}^j + I_t^j = R_tK_t^j\Delta + e^{-r_{ft}\Delta}L_{1t+\Delta}^j - L_t^j + Q_tI_t^j + Q_t\left(1-\delta\Delta\right)K_t^j
$$

$$
I_t^j \leq R_tK_t^j\Delta + e^{-r_{ft}\Delta}L_{1t+\Delta}^j - L_t^j,
$$

$$
V_{t+\Delta}(K_{1t+\Delta}^j, L_{1t+\Delta}^j) \geq V_{t+\Delta}\left(K_{1t+\Delta}^j, 0\right) - V_{t+\Delta}(\xi\left(1-\delta\Delta\right)K_t^j, 0),
$$

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